

Date _____ Week 12

Algebra

Subtopic: Polynomials

Lesson Topic: factorization of polynomials

INSTRUCTIONAL RESOURCES: A cardboard showing factor theorem

SPECIFIC OBJECTIVES:

Factorize polynomials of higher degree

Solve problems relating to polynomials

Find the roots of polynomial equations

Factor theorem:

A special case of the remainder theorem is when $f(x)$ leaves no remainder when divided by $x - a$.

We therefore say that $x - a$ is a factor of $f(x)$. The modified theorem is called factor theorem and it states that

If $f(a) = 0$, then $x - a$ is a factor of $f(x)$

STUDENTS ACTIVITIES: the students copy the notes

Solve:

Find the remainder when $f(x) = 2x^3 + 3x^2 - 4x + 1$ is divided by $2x - 1$ what conclusion can you draw?

Solution:

Let R be the remainder when $f(x)$ is divided by $2x - 1$, then $R = f\left(\frac{1}{2}\right)$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1$$

$$\Rightarrow \frac{1}{4} + \frac{3}{4} - 2 + 1 = 0$$

Hence: $2x - 1$ is a factor of $f(x)$

Solve:

Factorize $f(x) = x^3 - 7x^2 + 14x - 8$ completely

Solution:

± In a complete factorized form

$$f(x) = (x + p)(x + q)(x + r) \text{ -----}(i)$$

The first term of the expression of equation (i) is x^3 while the last term of the expression is $\pm pqr$. Hence p, q, r must be factor of -8 therefore try

$$x \pm 1$$

$$x \pm 2$$

$$x \pm 4$$

$$x \pm 8$$

$$f(-1) = 1 - 7 - 14 - 8 = -30 \neq 0$$

$x + 1$ is not a factor of $f(x)$

$$\begin{array}{r}
 x^2 - 6x + 8 \\
 x - 1 \overline{) x^3 - 7x^2 + 14x - 8} \\
 \underline{x^3 - x^2} \\
 -6x^2 + 14x \\
 \underline{-6x^2 + 6x} \\
 8x - 8 \\
 \underline{8x - 8} \\
 0
 \end{array}$$

$$\begin{aligned}
 \text{Now, } x^2 - 6x + 8 &= x^3 - 7x^2 + 14x - 8 \\
 &= (x - 2)(x - 4)
 \end{aligned}$$

$$\text{Hence: } f(x) = (x - 1)(x - 2)(x - 4)$$

Solve:

$$\text{Given that } f(x + 1) = x^2 + 3x - 1, \text{ find } f(3)$$

Solution:

$$\text{Put } x + 1 = 3$$

$$x = 2$$

$$\therefore f(x) = (2)^2 + 3(2) - 1$$

$$= 4 + 6 - 1 = 9$$

Solve:

$$\text{Factorize } x^2 - 5x + 6 = f(x)$$

Solution:

$$f(x) = x^2 - 5x + 6$$

$$f(2) = (2)^2 - 5(2) + 6$$

$$f(x) = 4 - 10 + 6 = 0$$

$f(2)$ is not $\Rightarrow (x - 2)$ is a factor

$$\begin{array}{r} x - 3 \\ x - 2 \overline{) x^2 - 5x + 6} \\ \underline{x^2 - 2x} \\ -3x + 6 \\ \underline{-3x + 6} \\ 0 \end{array}$$

$$f(x) = (x - 2)(x - 3)$$

ASSIGNMENT:

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